From Neutron Stars to Strange Stars

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1   Introduction

It is generally agreed that the tremendous densities reached in the centers of neutron stars provide a high pressure environment in which numerous particles processes are likely to compete with each other. These processes range from the generation of hyperons to quark deconfinement to the formation of kaon condensates and H-matter [1]. Another striking possibility concerns the formation of absolutely stable strange quark matter, a configuration of matter even more stable than the most stable atomic nucleus, iron. In the latter event all neutron stars would in fact be strange (quark matter) stars [2], objects largely composed of pure strange quark matter, eventually enveloped in a thin nuclear crust made up of ordinary, hadronic matter.

There has been much recent progress in our understanding of quark matter, culminating in the discovery that if quark matter exists it will be in a color superconducting state [3, 4, 5, 6]. The phase diagram of such matter appears to be very complex [5, 6]. At asymptotic densities the ground state of QCD with a vanishing strange quark mass is the color-flavor locked (CFL) phase. This phase is electrically neutral in bulk for a significant range of chemical potentials and strange quark masses [7]. If the strange quark mass is heavy enough to be ignored, then up and down quarks may pair in the two-flavor superconducting (2SC) phase. Other possible condensation patters are the recently discovered CFL–K^0 phase [8] and the color-spin locked (2SC+s) phase [9]. The magnitude of the gap energy lies between ~ 50 and 100 MeV. Color superconductivity thus modifies the equation of state (eos) at the order $(\Delta/\mu)^2$ level, which is only a few percent. Such small effects can be safely neglected in present determinations of models for the eos of neutron stars and strange quark matter stars. There has been much recent work on how color superconductivity in neutron stars could affect their properties [5, 6, 10, 11, 12]. These studies revealed that possible signatures include the cooling by neutrino emission, the pattern of the arrival times of supernova neutrinos, the evolution of neutron star magnetic fields, rotational (r-mode) instabilities, and glitches in rotation frequencies. In this review I shall complement this

list by reviewing several, most intruigung astrophysical implications connected with the possible absolute stability of strange quark matter. (Surface properties of strange matter are discussed in Usov's paper elsewhere in this volume.) This is followed by a discussion of two astrophysical signals that may point at the existence of quark matter in both isolated neutron stars as well as in neutron stars in low-mass x-ray binaries (LMXBs). We recall that a convincing discovery of quark matter in neutron stars would demonstrate that strange quark matter is not absolutely stable, ruling out the absolute stability of strange quark matter and the existence of strange quark stars, for it is not possible for neutron stars to contain quark matter cores and strange matte quark stars to both be stable [5].

2   Nuclear crusts on strange matter stars

Since stars in their lowest energy state are electrically charge neutral to very high precision, any net positive quark charge must be balanced by leptons. As a general feature, there is only very little need for leptons, since charge neutrality can be achieved essentially among the quarks themselves. (This is specifically the case for superconducting CFL quark matter in the asymptotic limit.) If electrons form a component of absolutely stable strange quark matter, their presence is crucial for the possible existence of a nuclear curst on such matter [13, 14]. The reason being that the electrons, which are bound to strange matter by the Coulomb force rather than the strong force, extend several hundred fermi beyond the surface of strange matter. Associated with this electron displacement is a very strong electric dipole layer which can support, out of contact with the surface of the strange matter, a crust of nuclear material, which it polarizes. The maximal possible density at the base of the crust (inner crust density) is determined by neutron drip, $\epsilon_{\rm drip} = 4.3 \times 10^{11}$ g/cm^3, at which neutrons begin to drip out of the nuclei and form a free neutron gas. Being electrically charge neutral, the neutrons do not feel the repulsive Coulomb force and hence would gravitate toward the quark matter core, where they become converted into strange matter. Neutron drip thus sets a strict upper limit on the crust's maximal inner density. The actual value may be slightly smaller though [15]. The somewhat complicated situation of the structure of a strange matter star with crust can be represented by a proper choice of eos [16], which consists of two parts. At densities below neutron drip it is represented by the low-density eos of charge-neutral nuclear matter, for which we use the Baym-Pethick-Sutherland eos. The star's strange matter core is described by the bag model.

3   Complete sequences of strange matter stars

Since the nuclear crusts surrounding the cores of strange stars are bound by the gravitational force rather than confinement, the mass-radius relationship of strange matter stars with crusts is qualitatively similar to the one of purely gravitationally bound stars, neutron stars and white dwarfs, as illustrated in Fig. 1 [17, 18]. The
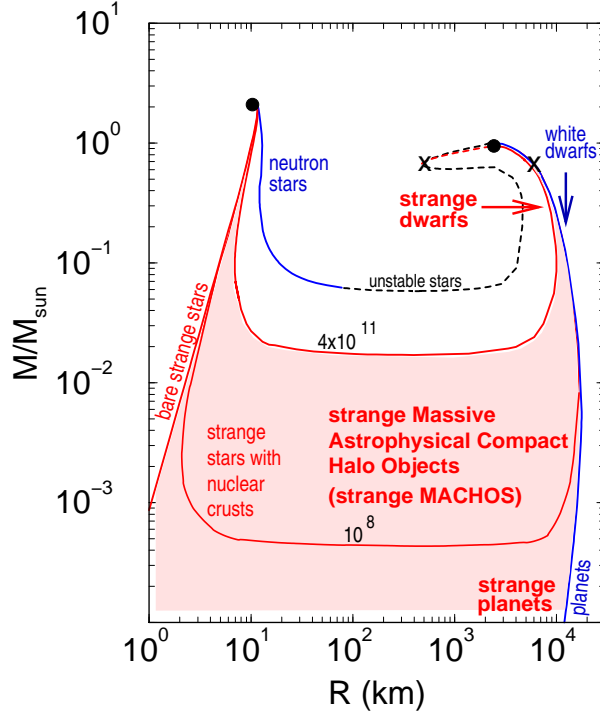

Figure 1: Classification of stellar objects in the mass-radius plane. Bare strange stars obey $M \propto R^3$. Nuclear crusts on them give way to an expansive range of novel stellar configurations (shaded area). These range from strange MACHOS to strange dwarfs to strange planets. The labels 10^8 and 4×10^{11} refer to inner crust densities in g/cm^3.

strange star sequences are computed for the maximal possible inner crust density, $\epsilon_{\mathrm{crust}} = \epsilon_{\mathrm{drip}}$, as well as for an arbitrarily chosen, smaller value of $\epsilon_{\mathrm{crust}} = 10^8$ g/cm^3, which may serves to demonstrate the influence of less dense crusts on the mass-radius relationship [17]. From the maximum mass star (dot in the upper left corner), the central density decreases monotonically through the sequence in each case. The stars located along the dashed line represent unstable configurations [17, 18]. The fact that strange stars with crusts tend to possess somewhat smaller radii than neutron stars implies smaller mass shedding (Kepler) periods P_{K} for the former. This is already indicated by the classical expression $P_{\mathrm{K}} = 2\pi\sqrt{R^3/M}$ and carries over to the full

general relativistic determination of P_K [1],

$$P_K = 2\pi \left(\omega + \frac{\omega'}{2\psi'} + e^{\nu-\psi}\sqrt{\frac{\nu'}{\psi'} + \left(\frac{\omega'}{2\psi'}e^{\psi-\nu}\right)^2} \right)^{-1}.$$ (1)

This expression is to be computed simultaneously in combination with Einstein's field equations for a rotating compact body [1],

$$R^{\kappa\lambda} - \frac{1}{2} g^{\kappa\lambda} R = 8\pi T^{\kappa\lambda}(\epsilon, P(\epsilon)).$$ (2)

It is found that, due to the smaller radii of strange stars, the complete sequence of such objects, and not just those close to the mass peak as is the case for neutron stars, can sustain extremely rapid rotation [17]. In particular, model calculations indicate for a strange star with a typical pulsar mass of $\sim 1.45\,M_\odot$ Kepler periods in the range of 0.55 msec $\lesssim P_K \lesssim 0.8$ msec, depending on the thickness of the nuclear curst and the bag constant [16, 17]. This range is to be compared with $P_K \sim 1$ msec obtained for neutron stars of the same mass.

The minimum-mass configurations of the sample strange star sequences in Fig. 1 have masses of about $\sim 0.017\,M_\odot$ (about 17 Jupiter masses) and $10^{-4}\,M_\odot$, depending on the valuer of ϵ_{crust}. For inner crust densities smaller than 10^8 g/cm^3 one obtains stable strange matter stars that can be by orders of magnitudes lighter than Jupiters. If abundant enough, these light strange stars could be seen by the gravitational microlensing searches [19]. Strange stars located to the right of the minimum mass configuration of each sequence consist of small strange cores, typically smaller than about 3 km, surrounded by nuclear crusts (ordinary white dwarf matter) that are thousands of kilometers thick. Such objects are called strange dwarfs. Their cores have shrunk to zero at the crossed points. What is left are ordinary white dwarfs with central densities equal to the inner crust densities of the former strange dwarfs. A stability analysis of strange stars against radial oscillations [17] shows that all strange dwarf sequences that terminate at stable ordinary white dwarfs are stable against radial oscillations. Strange stars that are located to the left of the mass peak of ordinary white dwarfs (solid dot in upper right corner), however, are unstable against oscillations and thus cannot exist in nature. So, in sharp contrast to neutron stars and white dwarfs, the branches of strange stars and strange dwarfs are stably connected with each other [17, 18]. Finally we would like to stress that strange dwarfs with 10^9 g/cm$^3 < \epsilon_{\text{crust}} < 4 \times 10^{11}$ g/cm^3 form entire new classes of stars that contain nuclear material up to $\sim 4 \times 10^4$ times denser than in ordinary white dwarfs of average mass, $M \sim 0.6\,M_\odot$ (central density $\sim 10^7$ g/cm^3). The entire family of such strange stars owes its stability to the strange core. Without the core they would be placed into the unstable region between ordinary white dwarfs and neutron stars [18].

Until recently, only rather vague tests of the theoretical mass-radius relation of white dwarfs have been possible. This has changed because of the availability of new

data emerging from the Hipparcos project [20]. These data allow the first accurate measurements of white dwarf distances and, as a result, establishing the mass-radius relation of such objects empirically. Figure 2 shows a comparison of several data from
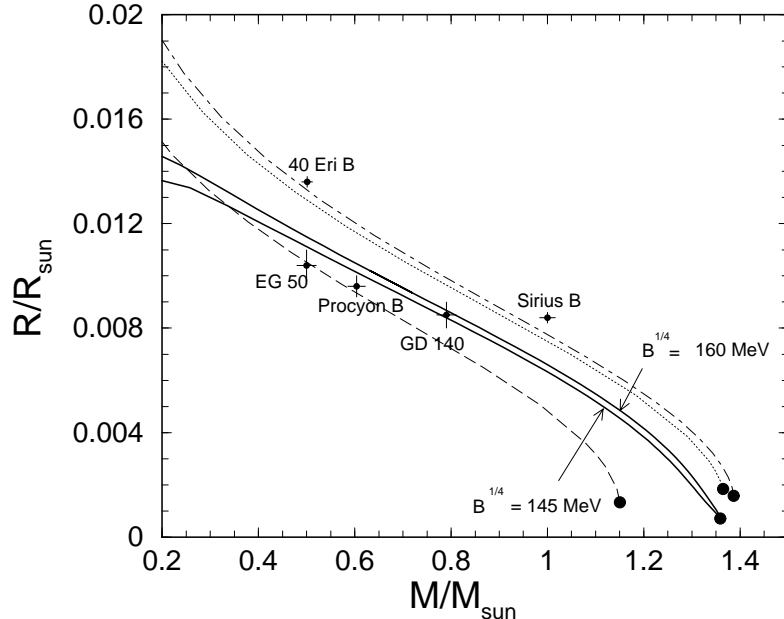

Figure 2: Comparison of theoretical mass-radius relationships of strange dwarfs ($\epsilon_{\text{crust}} = \epsilon_{\text{drip}}$) and several ordinary white dwarf stars [21] (^{12}C dwarfs: dot-dashed, ^{24}Mg: dotted, ^{56}Fe: dashed double-dotted) with data from the Hipparcos project.

the Hipparcos project with the mass-radius relationships of strange dwarfs (solid lines) and ordinary white dwarfs computed for different compositions.

4   Post-glitch behavior of strange stars

Ordinary neutron stars older than a few months have crusts made of a crystal lattice or an ordered inhomogeneous medium reaching from the surface down to regions with a density of 2×10^{14} g/cm^3. This crust contains only a few percent of the total moment of inertia. Strange stars, in contrast, can only support a crust with a density below neutron drip (4.3×10^{11} g/cm^3), for reasons discussed in section 2. Such a strange star crust contains at most $\sim 10^{-5}$ of the total moment of inertia. This is an upper bound, since the strange star may have no crust at all, depending on its prior evolution. The difference in the moment of inertia stored in the crust of neutron stars and strange stars seems to pose significant difficulties for explaining the glitch phenomenon observed in radio pulsars with models based on strange stars. Glitches

are observed as sudden speed-ups in the rotation rate of pulsars. The fractional change in rotation rate Ω is $\Delta\Omega/\Omega \approx 10^{-6} - 10^{-9}$, and the corresponding fractional change in the spin-down rate $\dot{\Omega}$ is of order $\Delta\dot{\Omega}/\dot{\Omega} \approx 10^{-2} - 10^{-3}$. In [16] it has been demonstrated that strange stars with crusts obey

$$\frac{\Delta\Omega}{\Omega} \simeq (10^{-5} \text{ to } 10^{-3}) \, f \,, \qquad (0 < f < 1) \,, \tag{3}$$

where f represents the fraction of the crustal moment of inertia that is altered in the quake, $|\Delta I| = f I_{\text{crust}}$. Since observed glitches have relative frequency changes of $\Delta\Omega/\Omega \simeq 10^{-9}$ to 10^{-6}, a change in the crustal moment of inertia by less than 10% would cause a giant glitch ($\Delta\Omega/\Omega \sim 10^{-6}$) even in the least favorable case. Of course there remains the question of whether there can be a sufficient build up of stress and also of the recoupling of crust and core which involves the healing of the pulsar period. This is probably a very complicated process that does not simply involve the recoupling of two homogeneous substances. For $\Delta\dot{\Omega}/\dot{\Omega}$ we established that [16]

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} > (10^{-1} \text{ to } 10) \, f \,, \tag{4}$$

yielding a small f value as before, namely $f < 10^{-4}$ to 10^{-1}. We have used measured values of the ratio $(\Delta\Omega/\Omega)/(\Delta\dot{\Omega}/\dot{\Omega}) \sim 10^{-6}$ to 10^{-4} for the Crab and Vela pulsars respectively. So the observed range of the fractional change in $\dot{\Omega}$ is consistent with the crust having the small moment of inertia calculated and the quake involving only a small fraction, f, of that, just as in Eq. (3). Nevertheless, without undertaking a study of whether the nuclear solid crust on strange stars could sustain a sufficient buildup of stress before cracking to account for such a sudden change in relative moment of inertia, or whether the healing-time and intervals between glitches can be understood, one cannot say definitely that strange stars with a nuclear solid crust can account for any complete set of glitch observations for a particular pulsar.

5   Formation of strange dwarfs

At present there is neither a well-studied model for the formation of hypothetical strange dwarfs, nor exists a study that determines their abundance in the universe. One possible scenario would be the formation of strange dwarfs from main sequence progenitors that have been contaminated with strange nuggets over their lifetimes. We recall that the capture of strange matter nuggets by main sequence stars is an inevitable consequence if strange matter were more stable than hadronic matter [22] because then the Galaxy would be filled with a flux of strange nuggets which would be acquired by every object they come into contact with, i.e. planets, neutron stars, white dwarfs, and main sequence stars. Naturally, due to the large radii of the latter,

they arise as ideal large-surface long integration time detectors for the strange matter flux. Nuggets that are accreted onto neutron stars and white dwarfs, however, never reach their centers, where the gravitational potential is largest, because they are stopped in the lattice close to the surface due to the large structural energy density there. This prevents such stars from building up a cores of strange matter. The situation is different for main sequence stars which are diffuse in comparison with neutron stars and white dwarfs. In this case the accreted nuggets may gravitate to the star's core, accumulate there and form a strange matter core that grows with time until the star's demise as a main sequence star. Another plausible mechanism has to do with primordial strange matter bodies. Such bodies of masses between 10^{-2} and $1\,M_\odot$ may have been formed in the early universe and survived to the present epoch [23]. Such objects will occasionally be captured by a main sequence star and form a significant core in a single and singular event. The core's baryon number, however, cannot be significantly larger than $\sim 5 \times 10^{31}\,(M/M_\odot)^{-1.8}$ where M is the star's mass. Otherwise a main sequence star is not capable of capturing the strange matter core [2, 24]. Finally we mention that in the very early evolution of the universe lumps of hot strange matter will evaporate nucleons which are plausibly gravitationally bound to the lump. The evaporation will continue until the quark matter has cooled sufficiently. Depending on the original baryon number of the quark lump, a strange star or dwarf, both with nuclear crusts will have been formed.

6   Quark matter in isolated neutron stars

Whether or not quark deconfinement occurs in neutron stars makes only very little difference to their static properties, such as the range of possible masses and radii, which renders the detection of quark matter in such objects extremely complicated. This turns out to be strikingly different for rotating neutron stars (i.e. pulsars) which develop quark matter cores in the course of spin-down. The reason being that as such stars spin down, because of the emission of magnetic dipole radiation and a wind of electron-positron pairs, they become more and more compressed. For some rotating neutron stars the mass and initial rotational frequency may be just such that the central density rises from below to above the critical density for dissolution of baryons into their quark constituents. This effects the star's moment of inertia dramatically [25], as shown in figure 3. Depending on the ratio at which quark and normal matter change with frequency, the moment of inertia can decrease very anomalously, and could even introduce an era of stellar spin-up lasting for $\sim 10^8$ years [25]. Since the dipole age of millisecond pulsars is about 10^9 years, we may roughly estimate that about 10% of the ~ 25 solitary millisecond pulsars presently known could be in the quark transition epoch and thus could be signaling the ongoing process of quark deconfinement! Changes in the moment of inertia reflect themselves in the braking
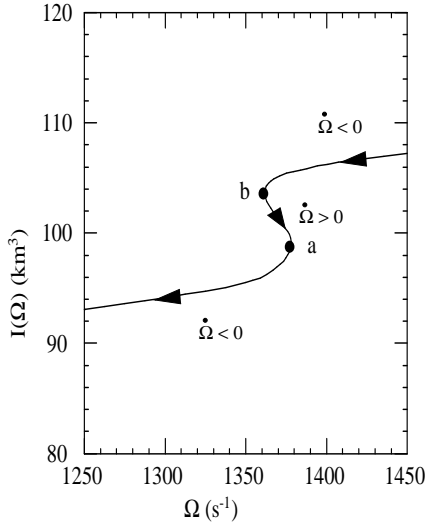
Figure 3: Moment of inertia versus frequency. The generation of quark matter causes a "backbending" of I for frequencies between a and b [1, 25].
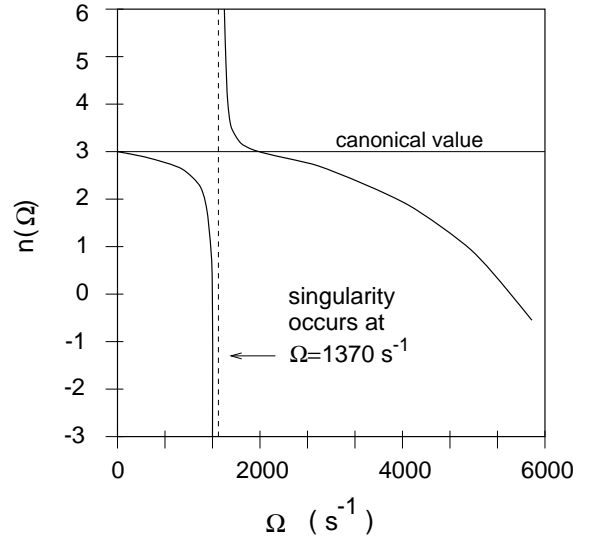

Figure 4: Anomaly in braking index caused by generation of quark matter.

index, n, of a rotating neutron star, as can be seen from ($I' \equiv \mathrm{d}I/\mathrm{d}\Omega$, $I'' \equiv \mathrm{d}^2 I/\mathrm{d}\Omega^2$)

$$n(\Omega) \equiv \frac{\Omega\,\ddot{\Omega}}{\dot{\Omega}^2} = 3 - \frac{3\,I'\,\Omega + I''\,\Omega^2}{2\,I + I'\,\Omega}. \tag{5}$$

The right-hand-side of this expression reduces to the well-known canonical constant $n = 3$ if I is independent of frequency. Evidently, this is not the case for rapidly rotating neutron stars, and it fails completely for stars that experience pronounced internal changes (phase transitions) which alter the moment of inertia significantly. Figure 4 illustrates this for the moment of inertia of the neutron star of Fig. 3. Because of the changes in I, caused by the gradual transition of hadronic matter into quark matter, the braking index deviates dramatically from 3 at the transition frequency, when pure quark matter is generated. Such dramatic anomalies in $n(\Omega)$ are not known for conventional neutron stars, because their moments of inertia appear to vary smoothly with Ω [1]. The future astrophysical observation of such anomalies in the braking behavior of pulsars may thus be interpreted as a signal for quark deconfinement in neutron stars.

7   Quark matter in accreting x-ray neutron stars

Accreting x-ray neutron stars provide a very interesting contrast to the spin-down of isolated neutron stars discussed in sect. 6. These x-ray neutron stars are being spun up
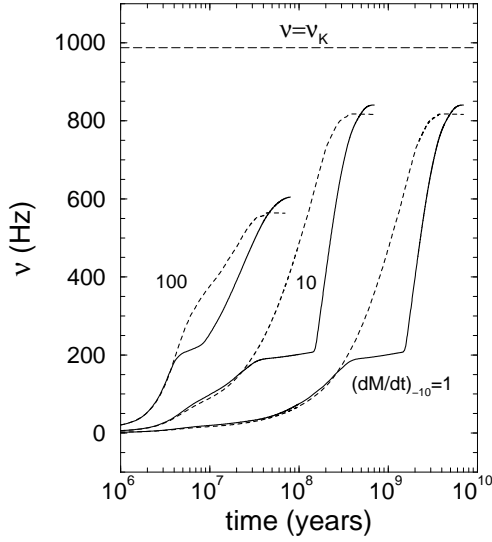
Figure 5: Evolution of spin frequencies of accreting x-ray neutron stars with (solid curves) and without (dashed curves) quark deconfinement [26]. The spin plateau around 200 Hz signals the ongoing process of quark reconfinement in the stellar centers [26].
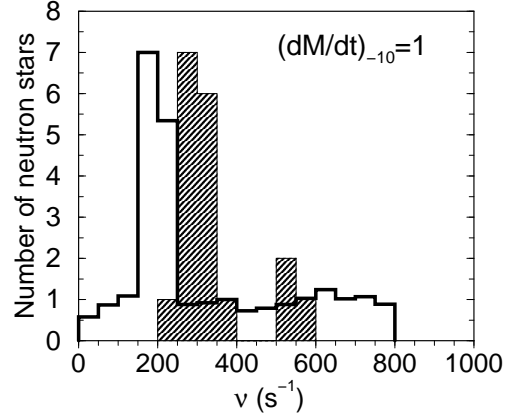
Figure 6: Calculated spin distribution of x-ray neutron stars. The spike in the calculated distribution (unshaded diagram) corresponds to the spinout of the quark matter phase. Otherwise the spike would be absent. The shaded histogram displays the observed data [26].

by the accretion of matter from a lower-mass ($M \lesssim 0.4 M_\odot$), less-dense companion. If the critical deconfinement density falls within that of canonical pulsars, quark matter will already exist in them but will be "spun out" of x-ray stars as their frequency increases during accretion. This scenario has been modeled in [26, 27] and will be discusses next. The spin-up torque experienced by a neutron star causes a change in the stars' angular momentum that is described by the relation

$$\frac{dJ}{dt} = \dot{M}\tilde{l}(r_{\rm m}) - N(r_{\rm c}),$$ (6)

where $\dot{M}$ denotes the accretion rate and $\tilde{l}(r_{\rm m}) = \sqrt{Mr_{\rm m}}$ is the angular momentum added to the star per unit mass of accreted matter. The quantity $N(r_{\rm c}) = \kappa\mu^2 r_{\rm c}^{-3}$ stands for the magnetic plus viscous torque terms, with $\mu \equiv R^3 B$ the star's magnetic moment. The quantities $r_{\rm m}$ and $r_{\rm c}$ denote the radius of the inner edge of the accretion disk and the co-rotating radius, respectively, and are given by ($\xi \sim 1$) $r_{\rm m} = \xi r_{\rm A}$ and $r_{\rm c} = (M\Omega^{-2})^{1/3}$. Accretion will be inhibited by a centrifugal barrier if the neutron star's magnetosphere rotates faster than the Kepler frequency at the magnetosphere. Hence $r_{\rm m} < r_{\rm c}$, otherwise accretion onto the star will cease. The Alfén radius $r_{\rm A}$, where the magnetic energy density equals the total kinetic energy of the accreting

matter is defined by $r_A = \mu^{4/7}(2M\dot{M}^2)^{-1/7}$. The rate of change of a star's angular frequency $\Omega = 2\pi/\nu \, (= J/I)$ then follows from Eq. (6) as

$$I(t)\frac{d\nu(t)}{dt} = \frac{\dot{M}\tilde{l}(t)}{2\pi} - \nu(t)\frac{dI(t)}{dt} - \kappa\,\mu(t)^2\,r_c(t)^{-3}\,, \tag{7}$$

with the explicit time dependences as indicated. Evidently, the second term on the right-hand-side of Eq. (7) depends linearly on Ω while the third terms grows quadratically with Ω. The temporal change of the moment of inertia of accreting neutron stars which undergo phase transitions is crucial [26, 27]. This also renders the calculation of the moment of inertia, given by [1]

$$I(t) = 2\pi \int_0^\pi d\theta \int_0^{R(\theta;t)} dr\, e^{\lambda+\mu+\nu+\psi} \frac{\epsilon+P}{e^{2\nu-2\psi}-(\omega-\Omega)^2} \frac{\Omega-\omega}{\Omega}\,, \tag{8}$$

very cumbersome, for each quantity on the right-hand-side varies accordingly during stellar spin-up. (We assume rigid body rotation, which renders the star's frequency Ω constant throughout the star.) The solution of equation (7) is shown in Fig. 6. The result is most striking. One sees that quark matter remains relatively dormant in the stellar core until the star has been spun up to frequencies at which the central density is about to drop below the threshold density at which quark matter exists. As known from Fig. 3, this manifests itself in a significant increase of the star's moment of inertia. The angular momentum added to a neutron star during this phase of evolution is therefore consumed by the star's expansion, inhibiting a further spin-up until the quark matter has been converted into a mixed phase of matter made up of hadrons and quarks. Such accreters, therefore, tend to spend a greater length of time in the critical frequencies than otherwise. There will be an anomalous number of accreters that appear at or near the same frequency, as shown in Fig. 6. This is what was found recently with the Rossi x-ray Timing Explorer (shaded area in Fig. 6). Quark deconfinement constitutes an attractive explanation for this anomaly [26, 27, 28], though alternative explanations were suggested too [29, 30].

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Discussion

S. Balberg (Hebrew University): The "anomalous" behavior in spin down (up) of an isolated (pair in a LMXB) neutron star attributed to the quark phase is really a feature of the equation of state, not of quark matter in itself. In other words, we need a relatively sharp feature in the equation of state to get a significant change in the moment of inertia, which would, in principle be due to other phenomena as well.

Weber: One needs a pronounced feature in the equation of state in order to get an anomaly in the breaking index (spin-up). Hyperons and meson condensates lead to features in the equation of state that seem to be way to week to cause such a striking anomaly [1]. A softening alone, as caused by hyperons and/or bosons, is not sufficient. What is really required is a softening followed by a gradual stiffening at higher densities which is naturally obtained for hadron-quark matter gradually compressed to higher densities.

H. Heiselberg (NORDITA): How does $M(R)$ look for your quark matter star?

Weber: Glendenning and Kettner demonstrated the existence of non-identical neutron star twins, depending on whether or not the low-density sequence of neutron stars terminates at central densities that fall close to the end of the mixed phase of quarks and hadrons [31].